1901001101030001
EXAMINATION NOVEMBER 2024
MASTER OF ARTS (MATHEMATICS) (PART - I) (EXTERNAL)
TOPOLOGY - LEVEL 3

[Time: As Per Schedule] **[Max. Marks:100]**

Instructions:
1. Fill up strictly the following details on your answer book
a. Name of the Examination: **MASTER OF ARTS(MATHEMATICS) (PART-1) (EXTERNAL)**
b. Name of the Subject: **TOPOLOGY – LEVEL 3**
c. Subject Code No: **1901001101030001**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.

Seat No:

Student's Signature

Q.1

1) Let *(X, d)* be a metric space and T be the collection of all the open sets of *(X, d)* then prove that T is a topology on *X*. **7**

2) Prove that the intersection of two topological spaces is also a topological space. **7**

3) Prove that every second countable space is separable. 6

OR

1) Prove that a metric space is sequentially compact if and only if it has the Bolzano-Weierstrass property. **7**

2) Do as directed: **7**

I. If Y is a subspace of a topological space X and Z is a subspace of a topological space Y, then show that Z is a subspace of X.

II. Show that every subspace of discrete topological space is discrete.

3) Prove that discrete topology on X is metrizable. 6

Q.2

1) Show that R^n and C^n are connected. **7**

2) Let X be a topological space. If $\{X_i\}$ is a non-empty class of compact subspaces of X each of which is closed, and if $\cap_i X_i$ is non-empty, show that $\cap_i X_i$ is a compact subspace of X. **7**

3) If a metric space X is complete and totally bounded, prove that X is compact. 6

OR

1) Prove that every sequentially compact metric space is totally bounded. **7**

2) Define perfect set. Show that a set is perfect if and only if it is closed and has no isolated points. **7**

3) Define Lebesgue number. Prove that every sequentially compact metric space is compact. **6**

Q.3 1) Let X be a topological space and A $\subseteq$ X, then prove that $X \epsilon \bar{A} \Leftrightarrow$ each neighbourhood of x intersects A. **7**

2) State and prove Lindelöf's theorem. **7**

3) Define countably compact space. Prove that a second countable space is countably compact if and only if it is compact. **6**

OR

1) Prove that every T_2 space is T_1 space but the converse is not true. **7**

2) Prove that in a Hausdorff space, any point and disjoint compact subspace can be separated by open sets, in the sense that they have disjoint neighbourhoods. **7**

3) Prove that the subspace of a regular space is regular. **6**

Q.4 1) Prove that every closed subspace of normal space is normal. **7**

2) Prove that the product of any non-empty class of connected space is connected. **6**

3) Define locally connected space. Let X be a locally connected space. If Y is an open subspace of X, show that each component of Y is open in X. In particular, each component of X is open. **7**

OR

1) Prove that the range of a continuous function defined on a connected space is an interval. **7**

2) Define totally disconnected space. Let X be a Hausdorff space. Show that X is totally disconnected if X has an open base whose sets are also closed. **6**

3) Prove that the product of any non-empty class of totally disconnected space is disconnected. **7**

Q.5 1) Define component of a topological space. Prove that each component of a topological space is closed. **7**

2) Define totally disconnected space. Show that the component of totally disconnected space are its points. **7**

3) Prove or disprove that a connected subspace of a locally connected space is locally connected if X is real line. **6**

OR

1) Prove that the product of any non-empty class of connected locally connected space is locally connected. **7**

2) What is totally disconnected? Show that every discrete space is totally disconnected. **7**

3) Prove that each component of a locally connected space is open. **6**

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